

Discretization methods for Engineering

Homework 1

Information: Go to the web page <http://www.mathworks.com/support/> and look for information on Matlab programming, including “webinars” if necessary.

Problem 1. Write a function `isprime(s)` that determines whether the integer `s` is prime or not. Then, using this function, and given two integers n and m , build a rectangular matrix of dimensions $n \times m$ holding the first nm prime numbers.

The solution must include:

- A printout of the code
- An explanation of what it does
- A printout of an example

Problem 2. In \mathbb{R}^N the following two norms are defined:

$$\|\mathbf{x}\|_\infty = \max_{1 \leq k \leq N} |x_k|, \quad \|\mathbf{x}\|_2 = \left(\sum_{k=1}^N x_k^2 \right)^{1/2}$$

Show that the two norms are equivalent.

Problem 3. Show that if a square matrix is injective, then it is bijective. Show also that a symmetric positive definite matrix is injective, and hence bijective.

Problem 4. (not mandatory) Prove that for any square matrix $\mathbf{A} : \mathbb{R}^N \rightarrow \mathbb{R}^N$,

$$\frac{1}{\|\mathbf{A}^{-1}\|} = \min_{\mathbf{u} \in \mathbb{R}^N} \max_{\mathbf{v} \in \mathbb{R}^M} \frac{|\mathbf{v} \cdot \mathbf{A}\mathbf{u}|}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

Due date: September 17, 2014