

Discretization methods for Engineering

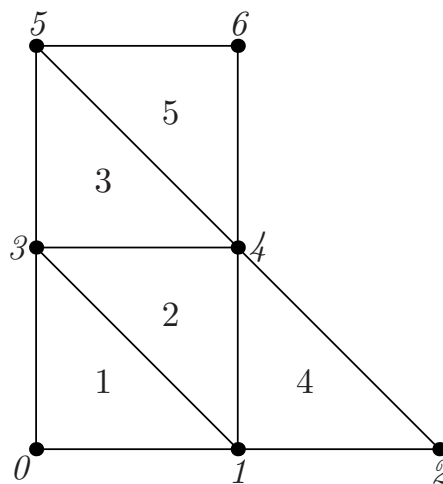
Assignment 3: The finite element method (due 23/10/13)

Problem 1. Calculate the integral

$$I = \int_0^{2\pi} \cos^2(x) dx$$

with Gauss quadratures of 1,2,3, and 4 points. Compare the results with the exact value.

Problem 2. Solve a thermal problem in the solid of the figure. All the elements in the solid are identical rectangular triangles with catheti of length 1 and conductivity $k = 3$. The base of the solid has an imposed temperature of value $\theta = 5$ and the edge 5 – 6 has also an imposed temperature of value $\theta = 10$. Finally, the point under node 4 has a fixed temperature of value $\theta = 20$. Find the solution with `matfem`, plotting the temperature profile and indicating the value of the thermal energy.



Subdivide elements 3 and 5, repeating the solution. Comment on the differences with respect to the first one.

Problem 3. Consider again the heat problem on the initial mesh of Problem 2.

- i) What is the total number of degrees of freedom in the problem (free+constrained)? And the number of free dofs? Print the value of the `id` matrix and interpret the results. Look at the function `solveproblem.m` and explain how this number is calculated.
- ii) Print the stiffness matrix `Kff` and the force vector `R`. What are the dimensions of these two quantities?
- iii) Look at the file `e1_thermal2D.m` which describes the equations that are being solved. What is the quadrature rule employed? Modify the heat source and obtain the solution for $r = \sin(y)$. Does this change the value of the internal force and the stiffness matrix? Why?